Zero-Forcing Equalizer based on Fast One Sided Jacobi Algorithm for MIMO PLC Systems

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Abstract

In this paper, we investigate the Zero-Forcing equalizer based on Fast One Sided Jacobi (ZF-FOSJ) algorithm for the Multiple Input Multiple Output Power Line Communication (MIMO PLC) systems. The proposed algorithm determines the pseudo-inverse of estimated channel matrix using only real matrix and operations free from trigonometric functions. The hardware implementation complexity is reduced compared to Zero-Forcing equalizer based on Two Sided Jacobi (ZF-TSJ) algorithm. The performance analysis of the ZF-FOSJ is based on hardware complexity and BER criteria.

Index Terms

Multiple Input Multiple Output (MIMO), Power Line Communications (PLC), Zero-Forcing (ZF), Fast One Sided Jacobi (FOSJ).

I. INTRODUCTION

The MIMO PLC system presents the new trend for high level communication \cite{1}, \cite{2}. In literature, various receiver architectures of MIMO equalizer are proposed. Generally, the performance improvement from on type of the MIMO equalizer to another comes at the price of higher hardware implementation costs. Despite of its simplicity, the ZF equalizer is known to suffer from the effect of noise enhancement \cite{3}.

One can enhance the ZF equalizer performance and reduce its hardware implementation costs by using the Singular Value Decomposition (SVD) based on Fast One Sided Jacobi (FOSJ) method. The FOSJ method is reputable for its ability to compute the singular values as well as left and right singular vectors with high relative accuracy.

As the FOSJ algorithm is proposed for real matrix. We are using at first, the real value decomposition for the estimated channel matrix. Then the FOSJ algorithm is applied into the real and the imaginary parts of the channel matrix for the computation of the pseudo-inverse.

The remainder of this paper is organized as follows; we present the system model in section II. In section III, we depict at first the real value decomposition of the channel matrix, and then we describe the MIMO PLC ZF equalizer based on FOSJ algorithm. We evaluate the performance of our proposal in section IV. Finally, we conclude this paper with section V.

II. SYSTEM MODEL

For a MIMO PLC system composed of \(N_t\) transmission ports and \(N_r\) reception ports, the MIMO channel can be described by a \(N_t \times N_r\) complex matrix \(H\). The MIMO PLC model is then given by

\[
\mathbf{r} = \mathbf{Hx} + \mathbf{n}
\]

Where \(\mathbf{x} = [x_1, x_2, \ldots, x_t]\) is the transmit signal, \(\mathbf{r} = [r_1, r_2, \ldots, r_r]\) is the received signal, and \(\mathbf{n} = [n_1, n_2, \ldots, n_r]\) is the noise at the receiver. The SVD form of the channel matrix is given by:

\[
\mathbf{H} = \mathbf{UDV}^H
\]

Where \(\mathbf{V}\) is the right-hand unitary matrix, \(\mathbf{U}\) is the left-hand unitary matrix, \(\mathbf{D}\) is the singular value matrix, and \((\cdot)^H\) is the Hermitian operation.
A. The One Sided Jacobi Algorithm

Invented by Hestenes, the One Sided Jacobi method consists of applying Jacobi rotations to orthogonalize two vectors. This is equivalent to the annihilation of matrix elements. The algorithm proceeds as follows. Let \( A \in \mathbb{R}^{m \times n} \) with \( m \geq n \), it consist at first to construct an orthogonal matrix \( V \) such that

\[
AV = \hat{U}
\]

(3)

Where the columns of \( \hat{U} \) are orthogonal. Normalizing the Euclidean length of each column of \( \hat{U} \) to unity, and setting \( \Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n) \) where \( \sigma_i \) is the 2-norm of the \( i \)-th column of \( \hat{U} \). So \( \hat{U} = U\Sigma \) and \( AV = U\Sigma \).

Based on the algorithm introduced in [4], for real symmetric matrix, the matrix \( V \) can be computed iteratively as a product of Givens rotations matrix [5]. Given a pair \( [a_i^k, a_j^k] \) of columns of the real matrix \( A \), we compute

\[
[a_i^{k+1}, a_j^{k+1}] = [a_i^k, a_j^k] \begin{pmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{pmatrix}
\]

(4)

The angle \( \theta \) is chosen to make \( a_i^{k+1} \) and \( a_j^{k+1} \) orthogonal to each other. The values of \( \cos(\theta) \) and \( \sin(\theta) \) are determined using Givens rotation. In general, the Givens rotation requires 4 multiplications and 2 additions for each iteration. Also it involves square roots operations, which is not desired when fixed point environment is established [6].

B. The One Sided Jacobi based on Fast-Given rotation

In order to reduce the complexity of computation of the parameters of Givens rotation, Tian in [7], proposed new plane of rotation free from calculating square roots and costs half as many multiplications compared to conventional Givens-rotation. The Fast Givens rotation \( G \) for input signal \( x = [x_1, x_2]' \) and output \( y = [y_1, y_2]' \) has the following formula:

\[
y = Gx = \begin{bmatrix}
\frac{x_1}{x_2} & 1 \\
-\frac{x_1}{x_2} & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = \begin{bmatrix}
x_1^2 + x_2^2 \\
0
\end{bmatrix}
\]

(5)

The Fast Givens rotation needs only 2 multiplications, 1 add, and 1 division. The One Sided Jacobi based on Fast Given rotation algorithm is nonenergy conservative but it can derives the pseudo-inverse of real matrix.

III. ZERO-FORCING EQUALIZER BASED ON FAST ONE SIDED JACOBI ALGORITHM

The proposed algorithm in [7], treats only real matrix. The generalization of such algorithm in the case of complex matrix, allow us to design new zero-forcing equalizer with low hardware complexity compared to the zero-forcing equalizer based on Two Sided Jacobi algorithms.

A. Real value Decomposition of Channel Matrix

Based on Real Value Decomposition the complex channel matrix with dimension \( m \times n \) can be decomposed into real bloc matrix with dimension \( 2m \times 2n \). The real value presentation of MIMO model can be written as

\[
y_R = \begin{bmatrix}
\Re[y] \\
\Im[y]
\end{bmatrix} = H_Rx_R + n_R
\]

(6)

Where \( \Re[x] \in \mathbb{R}^{n \times 1}, \Im[x] \in \mathbb{R}^{n \times 1}, \Re[n] \in \mathbb{R}^{n \times 1}, \Im[n] \in \mathbb{R}^{n \times 1} \). The complex channel matrix \( H \) is divided into 4 real blocs. In literature, for bloc defined matrix, if the diagonal blocs are invertible then the inverse of bloc defined matrix verifies \( H_R \times H_R^{-1} = H_R^{-1} \times H_R = I_{2n} \). Its proved that the inverse of the real value decomposition of channel matrix is given by

\[
H_R^{-1} = \begin{bmatrix}
\Re[H]^{-1} & \Im[H]^{-1} \\
\Im[H]^{-1} & \Re[H]^{-1}
\end{bmatrix}
\]

(7)

Where \( \Re[H]^{-1} = (\Re[H] + \Im[H]\Re[H]^{-1}\Im[H])^{-1} \) and \( \Im[H]^{-1} = \Im[H]^{-1}\Im[H]\Re[H]^{-1} \). We denote that the inverse of real value representation matrix has symmetric formula. So the computation of inverse bloc matrix is reduced to the half by calculating only two blocs of \( H_R^{-1} \). The two other blocs are deduced by applying symmetry, the blocs \( \Re[H]^{-1} \) and \( \Im[H]^{-1} \) are computed as a function of the real and the imaginary part of the channel matrix.
B. The Zero-Forcing Equalizer based on FOSJ

The generalization of FOSJ algorithm in case of complex matrix allows the conception of new architecture of zero forcing equalizer. This new form presents low complexity in hardware implementation and do not result in noise enhancement.

The zero-forcing module received the real and the imaginary parts of the estimated channel (see Fig.1) and applies transformations to obtain the real and the imaginary parts of the inverse of the channel matrix as described in the following Matlab algorithm:

Algorithm 1 Calculate $H^{-1}$

\[
\begin{align*}
R &= \Re[H] ; \\
I &= \Im[H] ; \\
R_{\text{inv}}^0 &= \text{OneSidedJacobi}(R) ; \\
A &= R + IR_{\text{inv}}^0 I ; \\
R_{\text{inv}}^1 &= \text{OneSidedJacobi}(A) ; \\
B &= R_{\text{inv}}^0 IR_{\text{inv}}^1 ; \\
H^{-1} &= R_{\text{inv}}^1 - iB ;
\end{align*}
\]

The proposed algorithm has simple structure and works only with real values to derive the inverse of complex matrix.

IV. SIMULATION RESULTS: REDUCTION IN COMPLEXITY WITHOUT DECREASING IN PERFORMANCE

A. Considerable gain in complexity

For the evaluation of the performance of the proposed algorithm, we are presenting in TABLE I the number of mathematical operation for the ZF equalizer based on FOSJ algorithm and the ZF equalizer based on TSJ algorithm for $2 \times 2$ MIMO systems.

<table>
<thead>
<tr>
<th>Operation</th>
<th>ZF-TSJ algorithm</th>
<th>ZF-FOSJ algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition/subtraction</td>
<td>84</td>
<td>46</td>
</tr>
<tr>
<td>Multiplication</td>
<td>154</td>
<td>104</td>
</tr>
<tr>
<td>Division</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>square root</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Cos/sin</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE I: mathematical operations numbers for the two algorithms
We denote that ZF-FOSJ presents reduced software complexity compared to the ZF-TSJ. Also, the FOSJ algorithm is free from trigonometric function (reduced hardware implementations costs).

B. Comparable performance to that of ZF-TSJ

We evaluate the performance of $2 \times 2$ MIMO PLC system based on ZF-FOSJ based on the Mean Bit Error Rate (Mean BER) criteria as a function of the Signal to Noise Ratio (SNR), for the frequency band $[0.5 MHz, 100 MHz]$. Simulations were carried out for 24 different MIMO channels generated and based on the MIMO PLC model presented in [8].

![Fig. 2: Comparable Mean BER of the proposed algorithm to that of the ZF-TSJ](image)

Figure 2 depicts that the Mean BER decrease considerably as a function of the SNR. It reaches a value lower than $10^{-5}$ for a $SNR = 20dB$. The simulation results prove that the Mean BER of proposed algorithm has similar behavior as the MIMO PLC system based on conventional ZF-TSJ algorithm.

V. Conclusion

In this paper we are presenting new architectures of ZF equalizer for MIMO PLC system based on FOSJ algorithm, adapted to hardware implementation. The developed algorithm presents reduced complexity keeping similar performance compared to ZF-TSJ algorithm.

REFERENCES


[2] D. Schneider, In home power line communications using multiple input multiple output principles, Dr.-Ing. dissertation, Verlag Dr. Hut, Munich, Germany, January 2012.


